

Stereotype bias: a simple formal model

François Bavaud

Department of Computer Science and Mathematical Methods
Department of Geography
University of Lausanne, Switzerland

Abstract

Minimizing the relative inertia of a statistical group with respect to the inertia of the overall sample defines an unique point, the in-focus, which constitutes a context-dependent measure of typical group tendency, biased in comparison to the group centroid. Maximizing the relative inertia yields an unique out-focal point, polarized in the reverse direction. This mechanism evokes the relative variability reduction of the outgroup reported in Social Psychology, and the stereotypic-like behavior of the in-focus, whose bias vanishes if the outgroup is constituted of a single individual. In this picture, the out-focus plays the role of an anti-stereotypical position, identical to the in-focus of the complementary group.

Keywords: anti-stereotype, central tendency, context-dependent polarization, Huygens principles, metacontrast ratio, outgroup homogeneity, relative dispersion, stereotype bias, variability reduction

1 Introduction

The expression “*typical features of a group*” is ambiguous: it might either refer to a multivariate indicator of group central tendency, that is to an unbiased *group centroid*, or to the distinctive, unique characteristic tendencies of the group, contrasting the features observed in the other groups or in the complete sample, in which case it constitutes a caricatural *stereotype*.

A stereotype summarizes the features of a whole group into a single profile (variability reduction). Its profile is generally distinct from the group centroid, pushed away from the overall centroid of the whole population or context under consideration (bias or polarization).

These two characteristics of stereotypy have been largely reported in Social Psychology, in particular with reference to the outgroup:

- 1) people tend to minimize the differences between outgroup members, while being inclined to perceive their own group as made of an heterogeneous set of unique individuals (*outgroup relative homogeneity*): “They all look alike but we don’t”. See e.g. Quattrone and Jones (1980), Taylor et al. (1978), Park and Hastie (1987), Mullen and Hu (1989), and references therein.
- 2) people tend to exaggerate the typical traits of the outgroup, and to enhance the contrasts between members of different groups (*stereotype polarization or bias effect*). See e.g. Turner (1975), Hopkins and Cable (2001), Hogg et al. (2004), Realo et al. (2009), and references therein.

This paper presents a simple, formal, principled mechanism linking the two aforementioned aspects of stereotypy. Specifically, we show that minimizing the *relative group dispersion* defines a unique point in the feature space, called the *in-focus* (Theorem 1), manifesting the exaggeration or polarization effect expected from a stereotype (Theorem 2). The polarization can be qualified as *fair*, in the sense it vanishes for a group formed of a single individual, which thus coincides with its own stereotype. Increasing the group dispersion increases the polarization effect, as shown by (4) and (6).

Furthermore, maximizing the relative group dispersion yields another unique point conjugate to the in-focus, the *out-focus*, bearing the characteristics of an anti-stereotype or *antitype*. The group out-focus coincides with the in-focus of the *complementary group* (Theorem 3), as illustrated on the U.S. Congressmen data (Section 2.4).

Section 3 attempts to justify the position of the in-focus in a decision-theoretical setup, and underlines the connections with the *meta-contrast* model and simulations of Salzarulo (2006), whose work initially triggered the present research.

2 The formal model

2.1 Definitions and notations: Huygens principle

Consider a totality n individuals, denoted $i = 1, \dots, n$, characterized by a multivariate profile of p features x_{ik} with $k = 1, \dots, p$. These features define

a squared Euclidean distance between individuals

$$D_{ij} = \sum_{k=1}^p (x_{ik} - x_{jk})^2 \quad i, j = 1, \dots, n \quad . \quad (1)$$

For generality sake, we assume that individuals possess *weights* $f_i > 0$, with $\sum_{i=1}^n f_i = 1$. The uniform weighting obtains as $f_i = 1/n$. Also, consider a profile $a \in \mathbb{R}^p$, which might or might not correspond to the features of an existing individual. Huygens principle consist of the identities

$$\Delta_f^a = \sum_{i=1}^n f_i D_{ia} = \Delta_f + D_{fa} \quad \Delta_f = \frac{1}{2} \sum_{ij} f_i f_j D_{ij} \quad (2)$$

where Δ_f^a is the inertia relative to the reference point a , D_{fa} is the squared distance between the point a and the centroid $\bar{x}_f = \sum_i f_i x_i$, and $\Delta_f = \Delta_f^{\bar{x}_f}$ is the inertia relative to the centroid. In particular, (2) shows that Δ_f^a attains its minimum Δ_f for $a = \bar{x}_f$.

Now consider a group g of individuals. A group is specified by the individuals it contains, that is, in full generality, by a distribution $g_i \geq 0$ with $\sum_{i=1}^n g_i = 1$. In most situations, the support of g (that is the set of individuals for which $g_i > 0$) is a strict subset of the complete set of the n individuals, but this restriction is not necessary. We however assume that the group centroid $\bar{x}_g = \sum_i g_i x_i$ differs from the overall centroid \bar{x}_f , that is $D_{fg} > 0$. As before, Δ_g^a takes on its minimum value Δ_g for $a = \bar{x}_g$.

2.2 The relative dispersion

Definition 1 (Relative dispersion) *The relative dispersion of group g in context f , relatively to the reference point a is*

$$\delta(a) = \delta(a|g, f) = \frac{\Delta_g^a}{\Delta_f^a} = \frac{\Delta_g + D_{ga}}{\Delta_f + D_{fa}} \quad . \quad (3)$$

The relative dispersion measures the disparity or heterogeneity in group g , in units determined by the overall heterogeneity, *as assessed from some reference point of view a* . Varying the point of view enables to tune, within some limits, the apparent, perceived relative heterogeneity of the group g . Remarkably enough, the relative dispersion $\delta(a)$ is finite everywhere, and possesses an unique minimum a_- as well as an unique maximum a_+ :

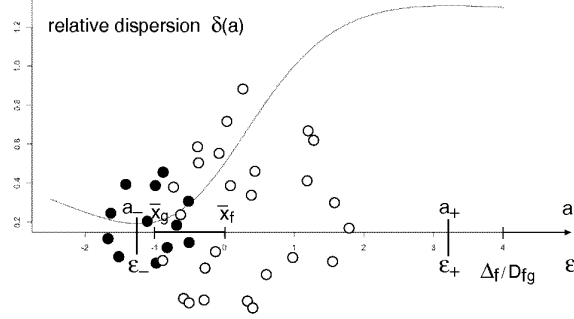


Figure 1: Abscissa: absolute coordinates a and relative coordinates ϵ , along the line (4) passing through the group centroid \bar{x}_g (associated to the black objects) and the overall centroid \bar{x}_f (associated to the black and white objects). Ordinate: the relative dispersion $\delta(a)$ is minimum for the in-focus points a_- , maximum for the out-focus points a_+ , and tends to unity as $\epsilon \rightarrow \pm\infty$.

Theorem 1 (In- and out-focus points) *Both the in-focus point a_- minimizing $\delta(a)$, and the out-focus point a_+ maximizing $\delta(a)$ are unique, and given by $a_{\pm} := a(\epsilon_{\pm})$, with*

$$a(\epsilon) := \bar{x}_f + \epsilon(\bar{x}_f - \bar{x}_g) \quad \epsilon_{\pm} = \frac{1}{2D_{fg}}(b_{fg} \pm \sqrt{b_{fg}^2 + 4\Delta_f D_{fg}}) \quad (4)$$

where $b_{fg} := \Delta_f - \Delta_g - D_{fg}$.

By theorem 1, the in- and out-focus points lie on the line joining centroids \bar{x}_f and \bar{x}_g . In-focus polarization occurs if a_- lies “on \bar{x}_g side”, that is if $\epsilon_- \leq -1$, as in Figure 1. Similarly, out-focus polarization occurs if a_+ lies “on \bar{x}_f side”, that is if $\epsilon_+ \geq 0$. Theorem 2 insures this to be always the case.

Theorem 2 (Polarization) *In- and out-focus points a_{\pm} fall outside the interval $[\bar{x}_g, \bar{x}_f]$, as in figure 1. Specifically,*

$$\epsilon_- \leq -1 \quad 0 < \epsilon_+ \leq \frac{\Delta_f}{D_{fg}} \quad (5)$$

where both inequalities are attained iff $\Delta_g = 0$, as in the case of a singleton,

dispersion-free group. For small Δ_g ,

$$\epsilon_- = -1 - \left[\frac{1}{\Delta_f + D_{fg}} \right] \Delta_g + 0(\Delta_g^2) \quad (6)$$

$$\epsilon_+ = \frac{\Delta_f}{D_{fg}} - \left[\frac{\Delta_f}{D_{fg}(\Delta_f + D_{fg})} \right] \Delta_g + 0(\Delta_g^2) \quad (7)$$

Proofs: let $e := \|\bar{x}_f - a\|$ be fixed, and consider the angle α between \bar{x}_g and a as measured from \bar{x}_f . By the cosine theorem,

$$\frac{\Delta_g + D_{ga}}{\Delta_f + D_{fa}} = \frac{\Delta_g + D_{fg} + e^2 - 2\sqrt{D_{fg}} e \cos \alpha}{\Delta_f + e^2}$$

which is maximum for $\alpha = 180^\circ$ and minimum for $\alpha = 0^\circ$. In both cases, the extremum a lies on the line passing through \bar{x}_g and \bar{x}_f , i.e. is of the form $a(\epsilon)$ in (4), with relative dispersion $(\Delta_g + D_{fg}(\epsilon + 1)^2)/(\Delta_f + D_{fg}\epsilon^2)$. Setting to zero its derivative in ϵ yields $D_{fg}\epsilon^2 - b_{fg}\epsilon - \Delta_f = 0$, with solutions ϵ_\pm (with the correct sign) given by (4). Furthermore, it is easy to show that, for D_{fg} and Δ_f fixed, both expressions ϵ_- and ϵ_+ are decreasing in Δ_g , and take on their maximum value (5) for $\Delta_g = 0$. \square

2.3 Other expressions

The following features-based expression may be computationally useful:

$$a_\pm = \sum_i \alpha_i(\epsilon_\pm) x_i \quad \alpha_i(\epsilon) := (1 + \epsilon)f_i - \epsilon g_i. \quad (8)$$

$\alpha(\epsilon)$ is a *signed* distribution, that is normalized to unity but not necessarily non-negative.

Also, twice application of Huygens decomposition (or direct manipulation of the features) demonstrates the distance-based identities

$$D_{fg} = -\frac{1}{2} \sum_{ij} (f_i - g_i)(f_j - g_j) D_{ij} \quad b_{fg} = \sum_{ij} f_i(f_j - g_j) D_{ij}.$$

Finally, define the *squared polarization ratio* as

$$\frac{D_{a-a+}}{D_{fg}} = (\epsilon_+ - \epsilon_-)^2 = \left(1 + \frac{\Delta_f + \Delta_g}{D_{fg}}\right)^2 - 4 \frac{\Delta_f \Delta_g}{D_{fg}^2} \geq 1$$

which shows the polarization to increase with each of the inertias Δ_f and Δ_g . In particular, the polarization ratio takes on its minimum value unity iff $\Delta_g = \Delta_f = 0$, and $a_- = \bar{x}_g$ iff $\Delta_g = 0$, as shown by (5).

2.4 Illustration: U.S. Congressmen

In the legislature 1984, a number of $p = 16$ “key votes” from $n = 435$ US Congressmen, comprising $n_R = 168$ Republicans and $n_D = 267$ Democrats, have been coded as 1 (“yea”) or 0 (“nay”)¹. Missing values have been replaced by the average value inside the affiliated political group.

The overall, Republican and Democrat distributions read respectively as

$$f_i = \frac{1}{n} \quad g_i = \frac{I(i \in R)}{n_R} \quad \bar{g}_i = \frac{I(i \in D)}{n_D} . \quad (9)$$

where $I(A)$ denotes the characteristic function of event A . After computation of the squared Euclidean distances (1) from the Congressmen votes, a classical multidimensional scaling (MDS) has been performed with uniform weighting of the individuals (see e.g. Mardia et al. 1979) to obtain the factorial coordinates expressing a maximum amount of the overall dispersion Δ_f (Figure 2). With $D_{fg} = 1.98$, $\Delta_f = 3.67$ and $\Delta_g = 1.89$, the polarization ratio is $|\epsilon_+ - \epsilon_-| = 2.72$.

2.5 Complementary group

By construction (Section 2.2), the in-focus a_- is the point of view under which group g appears, relatively to the *ground* or *context* formed by the complete set of individuals f , *as homogeneous as possible*, and turns out to constitute a credible candidate for representing a stereotypical value. By contrast, the out-focus a_+ is the point of view maximally respectful of the features diversity in g , and behaves as an *antitype* - in the sense of “anti-stereotypical”.

In the example of Figure 2, the out-focus a_+ of the Republicans seems to be equally qualified to represent the in-focus of the Democrats (not drawn on the Figure). As a matter of fact, the two points *coincide*, as justified by Definition 2 and Theorem 3 below.

Definition 2 *The group complementary to group g in context f is defined by a normed distribution \bar{g} , which, mixed with g , reproduces f , in the sense $\rho g + (1 - \rho)\bar{g} = f$, or equivalently*

$$\bar{g}_i = \frac{f_i - \rho g_i}{1 - \rho} \quad \text{for some } \rho \in (0, \rho_{\max}] \quad \text{with} \quad \rho_{\max} := \min_i (f_i / g_i) . \quad (10)$$

¹<http://archive.ics.uci.edu/ml/machine-learning-databases/voting-records/>

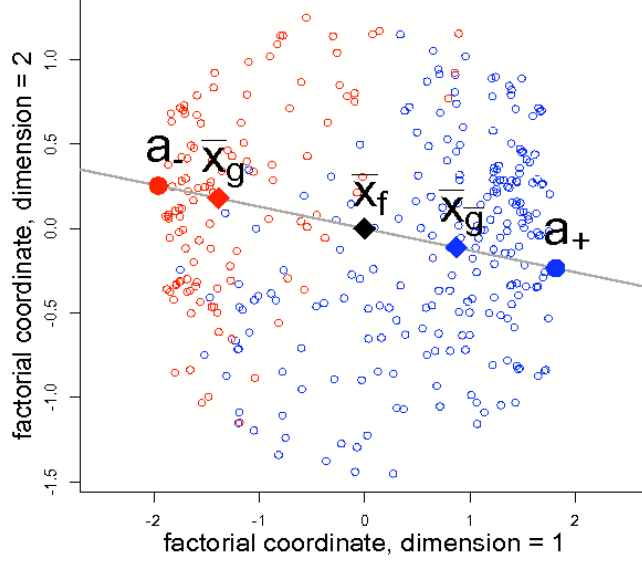


Figure 2: First MDS coordinates obtained from the squared distances between U.S. Congressmen (legislature 1984), expressing 58% of the overall dispersion Δ_f . Red circles depict the positions of the Republicans, and blue circles those of the Democrats. \bar{x}_f is the overall centroid, \bar{x}_g is the Republicans centroid, $\bar{x}_{\bar{g}}$ is the Democrats centroid. a_- is the in-focus relative to the Republicans, and a_+ the corresponding out-focus; see also Section 2.5.

The definition of ρ_{\max} insures the non-negativity of \bar{g} . For instance, the Democrats group \bar{g} in (9) is complementary to the Republicans group g since $\rho g + (1 - \rho)\bar{g} = f$ with $\rho = n_R/n = .39$, which turns out to be equal to its maximum value ρ_{\max} .

Theorem 3 *The out-focus point a_+^g for group g is the in-focus point $a_-^{\bar{g}}$ of any group \bar{g} complementary to g .*

Proof: substituting $f = \rho g + (1 - \rho)\bar{g}$ in (2) and developing the first identity demonstrates $\Delta_f^a = \rho(\Delta_g + D_{ga}) + (1 - \rho)(\Delta_{\bar{g}} + D_{\bar{g}a})$, that is $\rho \delta(a|g, f) + (1 - \rho) \delta(a|\bar{g}, f) = 1$. Hence, ρ , f and g being fixed, maximizing $\delta(a|g, f)$ amounts in minimizing $\delta(a|\bar{g}, f)$. \square

Note the identity

$$\delta'(a) = \delta(a|g, \bar{g}) = \frac{\Delta_g^a}{\Delta_{\bar{g}}^a} = \frac{1 - \rho}{\frac{\Delta_f^a}{\Delta_g^a} - \rho} \quad (11)$$

which demonstrates that extremalizing the relative dispersion $\delta(a) = \delta(a|g, f)$ or its variant $\delta'(a)$ yields the *same* solutions.

3 Further connections

3.1 Decision theory

The following argument constitutes a first attempt towards a derivation of the in-focus in a decision-theoretical framework. Consider the decision rule “*attribute individual i either to group g with probability $P(g|i) = \exp(-\beta D_{ia})$ or to the overall set f with probability $P(f|i) = 1 - P(g|i)$* ”, where $\beta > 0$ is a parameter controlling the decay of the exponential and a a point to be chosen wisely. The probability of misidentifying an individual of g (miss) is $P(f|g) = \sum_i g_i P(f|i)$, and the probability of correctly identifying an individual of f as such (correct rejection) is $P(f|f) = \sum_i f_i P(f|i)$. In the limit of large spread, the ratio of these quantities becomes

$$\lim_{\beta \rightarrow 0} \frac{P(\text{miss})}{P(\text{co.re.})} = \lim_{\beta \rightarrow 0} \frac{P(f|g)}{P(f|f)} = \lim_{\beta \rightarrow 0} \frac{\beta \sum_i g_i D_{ia} + 0(\beta^2)}{\beta \sum_i f_i D_{ia} + 0(\beta^2)} = \frac{\Delta_g^a}{\Delta_f^a} = \delta(a) \quad .$$

In this context, the probability of miss is minimized by the group centroid \bar{x}_g , while the ratio of the probabilities “miss over correct rejection” is minimized by the in-focus a_- - a somewhat intriguing result to be further investigated.

3.2 Subtractive combinations

Instead of studying the relative dispersion *ratio* (3), one can consider the *subtractive combination* of the form

$$\gamma(a) = A \Delta_g^a - B \Delta_f^a \quad A, B \in \mathbb{R} \quad . \quad (12)$$

If $A + B \neq 0$, the two parameters can be normalized as $A = 1 - \lambda$ and $B = \lambda$. For $\lambda \neq 0.5$, $\gamma(a)$ possesses a unique bounded extremum at $\epsilon = (\lambda - 1)/(1 - 2\lambda)$ (following parameterization (4)), which turns out to be a minimum for $\lambda < 0.5$ and a maximum for $\lambda > 0.5$; no bounded extremum exists for $\lambda = 0.5$. If $A + B = 0$, $\gamma(a)$ possesses a unique bounded extremum

at the mid-point $\epsilon = -0.5$, which constitutes a minimum for $A > 0$ and a maximum for $A < 0$. In any case, the position of the extremum does *not* depend upon the dispersions Δ_f and Δ_g . Similar results are obtained when replacing f by \bar{g} in (12).

3.3 Meta-contrast ratio and prototypicality function

More interesting, and considerably more involved is the study of the following function, appearing in the framework of the *self-categorization theory* (where groups are not given a priori), proposed by Salzarulo (2006), and referred to him (up to a sign) as the *prototypicality function*:

$$\Gamma(a) = (1 - \lambda) \Delta_{g(a)}^a - \lambda \Delta_{\bar{g}(a)}^a \quad \lambda \in [0, 1] \quad (13)$$

Here $g_i(a) = \exp(-\beta D_{ia})/Z(a)$, where $Z(a)$ is the normalization constant: in this approach, the very composition of group g depends on the distance of its constituents to a . Also, $\bar{g}(a)$ is of the form (10) with $\rho(a) = Z(a)/n < 1$.

The function (13) is primarily meant as an improved variant of the *meta-contrast ratio* (Haslam and Turner 1995; Turner et al. 1987; Oakes et al. 1994), measuring the relative differences between individuals, and aimed at predicting to which extent a given individual will be perceived as belonging to the subject group.

The highly non-linear properties of $\Gamma(a)$, whose minima are interpreted as prototypical positions, can be built on to run dynamical numerical simulations exhibiting groups formation and destruction, in the context of opinion formation, for various values of λ and β . In particular, two agents initially categorizing themselves as different can perceive themselves as belonging to the same group in presence of a third agent distant from them; also, fitting experimental data is possible, as those of Haslam and Turner (1995), satisfactorily reproduced with $\lambda = .08$ and $\beta = 7.7$, on a one-dimensional opinion space $x \in [0, 1]$. See Salzarulo (2006) for more details.

4 Discussion and conclusion

The mechanism relating the minimization of the relative dispersion to the polarization of the in-focus is entirely mathematical, and relevant to the construction of statistically biased, context-dependent measures of central (or “typical”) tendency. However, the parallel with a few predominant themes of Social Psychology seems striking, and we did not resist the temptation to interpret the in-focus as a stereotype, and the statistical group g as an out-group. The extent to which the metaphor is legitimate is to be judged within

Social Psychology. Among the points potentially stimulating, let us mention the question of the *identification of the ingroup*, of which both the context f and the complementary \bar{g} are legitimate candidates - with similar if not identical effects, in view of Theorem 3 and (11).

The formalism we have used is both general, that is using weighted groups allowing fuzzy memberships, and classical, that is using squared Euclidean distances as measures of dissimilarities. Euclidean distances permit, in contrast to other dissimilarities, to extract the original features through MDS (up to a rotation in the features space); they furthermore additively decompose accordingly to Huygens principles, the use of which has been crucial in the present paper.

Non-Euclidean dissimilarities, to which alternatives measures of central tendency are associated, such as the trimmed mean or the median (e.g. Hampel et al. 1986), are perfectly legitimate, and possibly better justified from robustness considerations. The resulting polarization of the in-focus would certainly deserve proper studies, which are however bound to be technically more involved.

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